Billings
PUBLIC SCHOOLS

# Mathematical Practice and Content

## Common Core Standards

# Geometry

March 2012

## PHILOSOPHY

**We believe every student can understand the general nature and uses of mathematics necessary to solve problems, reason inductively and deductively and apply numerical concepts necessary to function in a technological society. We believe instructional strategies must include real world applications and the appropriate use of technology. We believe students must be able to use mathematics as a communications medium.**

**Therefore, as an educational system we believe we can teach all children and all children can learn. We believe accessing knowledge, reasoning, questioning, and problem solving are the foundations for learning in an ever-changing world. We believe education enables students to recognize and strive for higher standards. Consequently, we will commit out efforts to help students acquire knowledge and attitudes considered valuable in order to develop their potential and/or their career and lifetime aspirations.**

## MATHEMATICAL PRACTICES

**The Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12.**

**1. Make sense of problems and persevere in solving them.**

***Mathematically proficient students:***

**a. Understand that mathematics is relevant when studied in a cultural context.**
**b. Explain the meaning of a problem and restate it in their words.**
**c. Analyze given information to develop possible strategies for solving the problem.**
**d. Identify and execute appropriate strategies to solve the problem.**
**e. Evaluate progress toward the solution and make revisions if necessary.**
**f. Check their answers using a different method, and continually ask "Does this make sense?"**

**2. Reason abstractly and quantitatively.**

***Mathematically proficient students:***

**a. Make sense of quantities and their relationships in problem situations.**
**b. Use varied representations and approaches when solving problems.**
**c. Know and flexibly use different properties of operations and objects.**
**d. Change perspectives, generate alternatives and consider different options.**

**3. Construct viable arguments and critique the reasoning of others.**

***Mathematically proficient students:***

**a. Understand and use prior learning in constructing arguments.**
**b. Habitually ask "why" and seek an answer to that question.**
**c. Question and problem-pose.**
**d. Develop questioning strategies to generate information.**
**e. Seek to understand alternative approaches suggested by others and. As a result, to adopt better approaches.**

**f. Justify their conclusions, communicate them to others, and respond to the arguments of others.**
**g. Compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.**

**4. Model with mathematics.**
***Mathematically proficient students:***
**a. Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. This includes solving problems within a cultural context, including those of Montana American Indians.**
**b. Make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.**
**c. Identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.**
**d. Analyze mathematical relationships to draw conclusions.**

**5. Use appropriate tools strategically.**
***Mathematically proficient students:***
**a. Use tools when solving a mathematical problem and to deepen their understanding of concepts (e.g., pencil and paper, physical models, geometric construction and measurement devices, graph paper, calculators, computer-based algebra or geometry systems.)**
**b. Make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. They detect possible errors by strategically using estimation and other mathematical knowledge.**

**6. Attend to precision.**
***Mathematically proficient students:***
**a. Communicate their understanding of mathematics to others.**
**b. Use clear definitions and state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.**
**c. Specify units of measure and use label parts of graphs and charts**
**d. Strive for accuracy.**

**7. Look for and make use of structure.**
***Mathematically proficient students:***
**a. Look for, develop, generalize and describe a pattern orally, symbolically, graphically and in written form.**
**b. Apply and discuss properties.**

**8. Look for and express regularity in repeated reasoning.**
***Mathematically proficient students:***
**a. Look for mathematically sound shortcuts.**
**b. Use repeated applications to generalize properties.**

## Traditional Pathway: Geometry

The fundamental purpose of this course in Geometry is to formalize and extend students' geometric experiences from the middle grades. Students explore more complex geometric situations and deepen their explanation of geometric relationships, moving towards formal mathematical arguments. Important differences exist between the Geometry course and the historical approach taken in Geometry classes. For example, transformations are emphasized early in this course. Close attention should be paid to the introductory content for the Geometry conceptual category found in the high school CCSS. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The critical areas, organized into six units are as follows.

**Critical Area/Unit 1:** In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence as a familiar foundation for the development of formal proof. Students prove theorems-using a variety of formats-and solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

**Critical Area/Unit 2:** Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean theorem. Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles, building on students' work with quadratic equations done in the first course. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles.

**Critical Area/Unit 3:** Students' experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line.

**Critical Area/Unit 4:** Building on their work with the Pythagorean theorem in $8^{th}$ grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines, which relates back to work done in the first course. Students continue their study of quadratics and algebraic definitions of the parabola.

**Critical Area/Unit 5:** In this unit, students prove basic theorems about circles, such as a tangent line in perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations, which relates back to work done in the first course, to determine intersections between lines and circles or parabolas and between two circles.

**Critical Area/Unit 6:** Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

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<tr><td>Unit 5:<br>Circles With and Without Coordinates</td><td>~Understand and apply theorems about circles<br>~Find arc lengths and areas of sectors of circles<br>~Translate between the geometric description and the equation for a conic section<br>~Use coordinates to prove simple geometric theorem algebraically<br>~Apply geometric concepts in modeling situations</td></tr>
<tr><td>Unit 6:<br>Applications of Probability</td><td>~Understand independence and conditional probability and use them to interpret data<br>~Use the rules of probability to compute probabilities of compound events in a uniform probability model<br>~Use probability to evaluate outcomes of decisions</td></tr>
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| Geometry | | | |
|---|---|---|---|
| Unit 1: Congruence, Proof, and Constructions | | | |
| Domains | Clusters with Instructional Notes | Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives | Mathematical Practices |
| Congruence | **Experiment with transformations in the plane.** *Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle.* | **G.CO.1 – Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.**<br>• Describe the terms point, line, and distance along a line in a plane<br>• Define perpendicular lines, parallel lines, line segments, and angles<br>• Define circle and the distance around a circular arc | 6. Attend to precision. |
| Congruence | **Experiment with transformations in the plane.** *Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle.* | **G.CO.2 – Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translations versus horizontal stretch).**<br>• Describe the different types of transformations including translations, reflections, rotations and dilations<br>• Describe transformations as functions that take points in the coordinate plane as inputs and give other points as outputs<br>• Write functions to represent transformations<br>• Compare transformations that preserve distance and angle to those that do not (e.g., translation vs. horizontal stretch) | 1. Make sense of problems and persevere in solving them.<br>4. Model with mathematics.<br>5. Use appropriate tools strategically.<br>6. Attend to precision.<br>8. Look for and express regularity in repeated reasoning. |
| Congruence | **Experiment with transformations in the plane.** *Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle.* | **G.CO.3 – Given a rectangle parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.**<br>• Describe rotations and reflections that carry a rectangle, parallelogram, trapezoid, or regular polygon onto itself. | 4. Model with mathematics.<br>6. Attend to precision.<br>7. Look for and make use of structure. |
| Congruence | **Experiment with transformations in the plane.** *Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle.* | **G.CO.4 – Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.**<br>• Recall definitions of angles, circles, perpendicular and parallel lines and line segments<br>• Define rotations, reflections, and translations | 3. Construct viable arguments and critique the reasoning of others. |

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| Congruence | **Experiment with transformations in the plane.** *Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle.* | **G.CO.5 – Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.**<br>• Given a geometric figure and a rotation, reflection or translation, draw the transformed figure<br>• Draw a transformed figure and specify the sequence of transformations that were used to carry the given figure onto the other | 3. Construct viable arguments and critique the reasoning of others.<br>5. Use appropriate tools strategically.<br>6. Attend to precision.<br>8. Look for and express regularity in repeated reasoning. |
| Congruence | **Understand congruence in terms of rigid motions.** *Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.* | **G.CO.6 – Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.**<br>• Determine if two figures are congruent using the definition of congruence in terms of rigid motions<br>• Use geometric descriptions of rigid motions to transform figures<br>• Use geometric descriptions of rigid motions to predict the effect of a given motion on a given figure | 3. Construct viable arguments and critique the reasoning of others.<br>5. Use appropriate tools strategically.<br>6. Attend to precision.<br>8. Look for and express regularity in repeated reasoning. |
| Congruence | **Understand congruence in terms of rigid motions.** *Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.* | **G.CO.7 – Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.**<br>• Identify corresponding angles and sides of two triangles<br>• Identify corresponding pairs of angles and sides of congruent triangles after rigid motions<br>• Justify congruency of two triangles using transformations<br>• Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if corresponding pairs of sides and corresponding pairs of angles are congruent<br>• Use the definition of congruence in terms of rigid motions to show that if the corresponding pairs of side and corresponding pairs of angles of two triangles are congruent, then the two triangles are congruent | 3. Construct viable arguments and critique the reasoning of others. |

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| Congruence | **Understand congruence in terms of rigid motions.** *Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.* | **G.CO.8 – Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.**<br>• Informally use rigid motions to take angles to angles and segments to segments (from 8th grade)<br>• Formally use dynamic geometry software or straightedge and compass to take angles to angles and segments to segments<br>• Explain how the criteria for triangle congruence (ASA, SAS, SSS) follows from the definition of congruence in terms of rigid motions | 1. Make sense of problems and persevere in solving them.<br>3. Construct viable arguments and critique the reasoning of others.<br>7. Look for and make use of structure.<br>8. Look for and express regularity in repeated reasoning. |
| Congruence | **Prove geometric theorems.** *Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.* | **G.CO.9 – Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.**<br>• Identify and use properties of vertical angles, parallel lines with transversals, all angle relationships, corresponding angles, alternate interior angles, perpendicular bisector, equidistant from endpoint<br>• Prove vertical angles are congruent<br>• Prove corresponding angles are congruent when two parallel lines are cut by a transversal and converse<br>• Prove alternate interior angles are congruent when two parallel lines are cut by a transversal and converse<br>• Prove points on a perpendicular bisector of a line segment are exactly equidistant from the segments endpoint | 1. Make sense of problems and persevere in solving them.<br>2. Reason abstractly and quantitatively.<br>3. Construct viable arguments and critique the reasoning of others.<br>6. Attend to precision.<br>7. Look for and make use of structure.<br>8. Look for and express regularity in repeated reasoning. |
| Congruence | **Prove geometric theorems.** *Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. G.CO.10 may be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for G.C.3 in Unit 5.* | **G.CO.10 – Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.**<br>• Identify the hypothesis and conclusion of a theorem<br>• Analyze components of a theorem<br>• Prove theorems about triangles<br>• Design an argument to prove theorems about triangles | 1. Make sense of problems and persevere in solving them.<br>3. Construct viable arguments and critique the reasoning of others.<br>7. Look for and make use of structure.<br>8. Look for and express regularity in repeated reasoning. |

| Geometry | | | |
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| Unit 1: Congruence, Proof, and Constructions | | | |
| Domains | Clusters with Instructional Notes | Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives | Mathematical Practices |
| Congruence | **Prove geometric theorems.** *Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.* | **G.CO.11 – Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.**<br>• Classify types of quadrilaterals<br>• Explain theorems for parallelograms and relate a figure<br>• Use the principle that corresponding parts of congruent triangles are congruent to solve problems<br>• Use properties of special quadrilaterals in a proof | 1. Make sense of problems and persevere in solving them.<br>3. Construct viable arguments and critique the reasoning of others.<br>7. Look for and make use of structure. |
| Congruence | **Make geometric constructions.** *Build on prior student experience with simple constructions. Emphasize the ability to formalize and explain how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.* | **G.CO.12 – Make formal geometric constructions, including those representing Montana American Indians, with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment, copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.**<br>• Explain the construction of geometric figures using a variety of tools and methods<br>• Apply the definitions, properties, and theorems about line segments, rays, and angles to support geometric constructions<br>• Apply properties and theorems about parallel and perpendicular lines to support constructions<br>• Perform geometric constructions using a variety of tools and methods, including: Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line | 4. Model with mathematics.<br>5. Use appropriate tools strategically.<br>6. Attend to precision. |
| Congruence | **Make geometric constructions.** *Build on prior student experience with simple constructions. Emphasize the ability to formalize and explain how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.* | **G.CO.13 – Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.**<br>• Construct an equilateral triangle, square, and regular hexagon inscribed in a circle | 4. Model with mathematics. |

| Geometry | | | |
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| Unit 2: Linear and Exponential Relationships | | | |
| Domains | Clusters with Instructional Notes | Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives | Mathematical Practices |
| Similarity, Right Triangles, and Trigonometry | **Understand similarity in terms of similarity transformations.** | **G.SRT.1 – Verify experimentally the properties of dilations given by a center and a scale factor:**<br>**a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.**<br>**b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.**<br>• Define image, pre-image, scale factor, center, and similar figures as they relate to transformations<br>• Identify a dilation starting its scale factor and center<br>• Explain that the scale factor represents how many times longer or shorter a dilated line segment is than its pre-image<br>• Verify experimentally that the dilation of a line segment is longer or shorter in the ratio given by the scale factor<br>• Verify experimentally that a dilated image is similar to its pre-image by showing congruent corresponding angles and proportional sides<br>• Verify experimentally that a dilation takes a line not passing through the center of the dilation to a parallel line by showing the lines are parallel<br>• Verify experimentally that dilation eaves a line passing through the center of the dilation unchanged, by showing that it is the same line | 1. Make sense of problems and persevere in solving them.<br>2. Reason abstractly and quantitatively.<br>3. Construct viable arguments and critique the reasoning of others.<br>7. Look for and make use of structure.<br>8. Look for and express regularity in repeated reasoning. |
| Similarity, Right Triangles, and Trigonometry | **Understand similarity in terms of similarity transformations.** | **G.SRT.2 – Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.**<br>• Given two figures, decide if they are similar by using the definition of similarity in terms of similarity transformations | 1. Make sense of problems and persevere in solving them.<br>3. Construct viable arguments and critique the reasoning of others.<br>6. Attend to precision. |
| Similarity, Right Triangles, and Trigonometry | **Understand similarity in terms of similarity transformations.** | **G.SRT.3 – Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.**<br>• Recall the properties of similarity transformations<br>• Establish the AA criterion for similarity of triangles by extending the properties of similarity transformations to the general case of any two similar triangles | 1. Make sense of problems and persevere in solving them.<br>3. Construct viable arguments and critique the reasoning of others. |
| Similarity, Right Triangles, and Trigonometry | **Prove theorems involving similarity.** | **G.SRT.4 – Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.**<br>• Recall postulates, theorems, and definitions to prove theorems about triangles<br>• Prove theorems involving similarity about triangles | 3. Construct viable arguments and critique the reasoning of others. |

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| Unit 2: Linear and Exponential Relationships | | | |
| Domains | Clusters with Instructional Notes | Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives | Mathematical Practices |
| Similarity, Right Triangles, and Trigonometry | **Prove theorems involving similarity** | **G.SRT.5 – Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.**<br>• Recall congruence and similarity criteria for triangles<br>• Use congruency and similarity theorems for triangles to solve problems<br>• Use congruency and similarity theorems for triangles to prove relationships in geometric figures | 1. Make sense of problems and persevere in solving them.<br>3. Construct viable arguments and critique the reasoning of others. |
| Similarity, Right Triangles, and Trigonometry | **Define trigonometric rations and solve problems involving right triangles.** | **G.SRT.6 – Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.**<br>• Name the sides of right triangles as related to an acute angle<br>• Recognize that if two right triangles have a pair of acute, congruent angles, that the triangles are similar<br>• Compare common ratios for similar right triangles and develop a relationship between the ratio and the acute angle leading to the trigonometry ratios | 3. Construct viable arguments and critique the reasoning of others.<br>8. Look for and express regularity in repeated reasoning. |
| Similarity, Right Triangles, and Trigonometry | **Define trigonometric rations and solve problems involving right triangles.** | **G.SRT.7 – Explain and use the relationship between the sine and cosine of complementary angles.**<br>• Explain how the sine and cosine of complementary angles are related to each other<br>• Use the relationship between the sine and cosine of complementary angles | 3. Construct viable arguments and critique the reasoning of others.<br>8. Look for and express regularity in repeated reasoning. |
| Similarity, Right Triangles, and Trigonometry | **Define trigonometric rations and solve problems involving right triangles.** | **G.SRT.8 – Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.**<br>• Recognize which methods could be used to solve right triangles in applied problems<br>• Solve for an unknown angle or side of a right triangle using sine, cosine, and tangent<br>• Apply right triangle trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems | 4. Model with mathematics.<br>7. Look for and make use of structure. |
| Modeling with Geometry | **Apply geometric concepts in modeling situations.** *Focus on situations well modeled by trigonometric rations for acute angles.* | **G.MG.1 – Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder; modeling a Montana American Indian tipi as a cone).**<br>• Given a real world object, classify the object as a known geometric shape – use this to solve problems in context<br>• Focus on situations well modeled by trigonometric ratios for acute angles<br>• Use measure and properties of geometric shapes to describe real world objects | 4. Model with mathematics.<br>7. Look for and make sense of structure. |
| Modeling with Geometry | **Apply geometric concepts in modeling situations.** *Focus on situations well modeled by trigonometric rations for acute angles.* | **G.MG.2 – Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTU's per cubic foot).**<br>• Define density<br>• Apply concepts of density based on area and volume to model real-life situations | 4. Model with mathematics.<br>7. Look for and make sense of structure. |

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| Unit 2: Linear and Exponential Relationships | | | |
| Domains | Clusters with Instructional Notes | Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives | Mathematical Practices |
| Modeling with Geometry | **Apply geometric concepts in modeling situations.** *Focus on situations well modeled by trigonometric rations for acute angles.* | **G.MG.3 – Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).**<br>• Describe a typographical grid system<br>• Apply geometric methods to solve design problems(e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios) | 1. Make sense of problems and persevere in solving them.<br>4. Model with mathematics. |
| Similarity, Right Triangles, and Trigonometry | **Apply trigonometry to general triangles.** *With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles.* | **G.SRT.9 – (+) Derive the formula A = ½ ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.**<br>• Recall right triangle trigonometry to solve mathematical problems<br>• Derive the formula A = ½ ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side | 4. Model with mathematics.<br>7. Look for and make use of structure. |
| Similarity, Right Triangles, and Trigonometry | **Apply trigonometry to general triangles.** *With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles.* | **G.SRT.10 – (+) Prove the Laws of Sines and Cosines and use them to solve problems.**<br>• Prove the Laws of Sines<br>• Prove the Law of Cosines<br>• Recognize when the Law of Sines or Law of Cosines can be applied to a problem and solve problems in context using them<br>• Use the Laws of Sines and Cosines to find missing angles or side length measurements | 1. Make sense of problems and persevere in solving them.<br>3. Construct viable arguments and critique the reasoning to others.<br>6. Attend to precision. |
| Similarity, Right Triangles, and Trigonometry | **Apply trigonometry to general triangles.** *With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles.* | **G.SRT.11 – (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).**<br>**• Determine from given measurements in right and non-right triangles whether it is appropriate to use** the Law of Sines or Cosines<br>• Apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces) | 1. Make sense of problems and persevere in solving them.<br>3. Construct viable arguments and critique the reasoning of others.<br>6. Attend to precision. |

| Geometry | | | |
|---|---|---|---|
| Unit 3: Extending to Three Dimensions | | | |
| Domains | Clusters with Instructional Notes | Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives | Mathematical Practices |
| Geometric Measurement and Dimension | **Explain volume formulas and use them to solve problems.** *Informal arguments for areas and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is $k^2$ times the area of the first. Similarly, volumes of solid figures scale by $k^3$ under a similarity transformation with scale factor k.* | **G.GMD.1 – Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments; Cavalieri's principle, and informal limit arguments.**<br>• Give an informal argument for the formulas for the circumference and area of a circle<br>• Given an informal argument for the formulas for the volume of a cylinder, pyramid, and cone<br>• Use dissection arguments, Cavalieri's principle, and informal limit arguments | 2. Reason abstractly and quantitatively. |
| Geometric Measurement and Dimension | **Explain volume formulas and use them to solve problems.** *Informal arguments for areas and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is $k^2$ times the area of the first. Similarly, volumes of solid figures scale by $k^3$ under a similarity transformation with scale factor k.* | **G.GMD.3 – Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.**<br>• Utilize the appropriate formula for volume, depending on the figure<br>• Use volume formulas for cylinders, pyramids, cones, and spheres to solve contextual problems | 4. Model with mathematics.<br>6. Attend to precision. |
| Geometric Measurement and Dimension | **Visualize the relation between two-dimensional and three-dimensional objects.** | **G.GMD.4 – Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.**<br>• Relate the shapes of two-dimensional cross-sections to their three-dimensional objects<br>• Discover three-dimensional objects generated by rotations of two-dimensional objects<br>• Use strategies to help visualize relationships between two-dimensional and three-dimensional objects | 2. Reason abstractly and quantitatively.<br>4. Model with mathematics. |

| Geometry | | | |
|---|---|---|---|
| Unit 3: Extending to Three Dimensions | | | |
| Domains | Clusters with Instructional Notes | Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives | Mathematical Practices |
| Modeling with Geometry | **Apply geometric concepts in modeling situations.** *Focus on situations that require relating two- and three-dimensional objects, determining and using volume, and the trigonometry of general triangles.* | **G.MG.1 – Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).**<br>• Given a real world object, classify the object as a known geometric shape – use this to solve problems in context<br>• Focus on situations well modeled by trigonometric ratios for acute angles<br>• Use measure and properties of geometric shapes to describe real world objects | 4. Model with mathematics.<br>7. Look for and make sense of structure. |

| Geometry | | | |
|---|---|---|---|
| Unit 4: Connecting Algebra and Geomtery Through Coordinates | | | |
| Domains | Clusters with Instructional Notes | Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives | Mathematical Practices |
| Expressing Geometric Properties with Equations | **Use coordinates to prove simple geometric theorems algebraically.** *This unit has a close connection with the next unit. For example, a curriculum might merge G.GPE.1 and the Unit 5 treatment of G.GPR.4 with the standards in this unit. Reasoning with triangles in this unit is limited to right triangles; e.g., derive the equation for a line through two points using similar right triangles.* | **G.GPE.4 – Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, the square root of 3) lies on the circle centered at the origin and containing the point (0, 2).**<br>• Recall previous understandings of coordinate geometry including distance, midpoint, and slope formulas, equation of a line, and definitions of parallel and perpendicular lines<br>• Use coordinates to prove simple geometric theorems algebraically<br>• Derive the equation of a line through 2 points using similar right triangles<br>• Derive simple proofs involving circles | 1. Make sense of problems and persevere in solving them.<br>2. Reason abstractly and quantitatively.<br>3. Construct viable arguments and critique the reasoning of others.<br>6. Attend to precision. |
| Expressing Geometric Properties with Equations | **Use coordinates to prove simple geometric theorems algebraically.** *This unit has a close connection with the next unit. Relate work on parallel lines in G.GPE.5 to work on A.REI.5 in Algebra I involving systems of equations having no solution or infinitely many solutions.* | **G.GPE.5 – Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).**<br>• Recognize that slopes of parallel lines are equal<br>• Recognize that slopes of perpendicular lines are opposite reciprocals<br>• Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems<br>• Find the equation of a line parallel to a given line that passes through a given point<br>• Find the equation of a line perpendicular to a given line that passes through a given point | 2. Reason abstractly and quantitatively.<br>3. Construct viable arguments and critique the reasoning of others. |
| Expressing Geometric Properties with Equations | **Use coordinates to prove simple geometric theorems algebraically.** *This unit has a close connection with the next unit.* | **G.GPE.6 – Find the point on a directed line segment between two given points that partitions the segment in a given ratio.**<br>• Recall the definition of ratio<br>• Recall previous understandings of coordinate geometry<br>• Given a line segment (including those with positive and negative slopes) and a ratio, find the point on the segment that partitions the segment into the given ratio | 2. Reason abstractly and quantitatively. |
| Expressing Geometric Properties with Equations | **Use coordinates to prove simple geometric theorems algebraically.** *This unit has a close connection with the next unit. G.GPE.7 provides practice with the distance formula and its connection with the Pythagorean Theorem.* | **G.GPE.7 – Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. ★**<br>• Use the coordinates of the vertices of a polygon to find the necessary dimensions for finding the perimeter<br>• Use the coordinates of the vertices of a triangle to find the necessary dimensions (base, height) for finding the area<br>• Use the coordinates of the vertices of a rectangle to find the necessary dimensions (base, height) for finding the area<br>• Formulate a model of figures in contextual problems to compute area and/or perimeter | 2. Reason abstractly and quantitatively. |
| Reasoning with Equations and Inequalities | **Translate between geometric description and the equation for a conic section.** *The directrix should be parallel to a coordinate axis.* | G.GPE.2 – Derive the equation of a parabola given a focus and directrix.<br>• Define a parabola including the relationship of the focus and the equation of the directrix to the parabolic shape<br>• Derive the equation of a parabola given the focus and directrix | 4. Model with mathematics.<br>7. Look for and make use of structure. |

| Geometry | | | |
|---|---|---|---|
| Unit 5: Circles with and without Coordinates | | | |
| Domains | Clusters with Instructional Notes | Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives | Mathematical Practices |
| Circles | **Understand and apply theorems about circles.** | **G.C.1 – Prove that all circles are similar.**<br>• Recognize when figures are similar<br>• Compare the ratio of the circumference of a circle to the diameter of the circle<br>• Discuss, develop, and justify the ratio of the circumference of a circle to the diameter of the circle for several circles | 5. Use appropriate tools strategically.<br>8. Look for and express regularity in repeated reasoning. |
| Circles | **Understand and apply theorems about circles.** | **G.C.2 – Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.**<br>• Identify inscribed angles, radii, chords, central angles, circumscribed angles, diameter, tangent<br>• Recognize that inscribed angles on a diameter are right angles<br>• Recognize that the radius of a circle is perpendicular to the radius at the point of tangency<br>• Examine the relationship between central, inscribed, and circumscribed angles by applying theorems about their measures | 2. Reason abstractly and quantitatively.<br>7. Look for and make use of structure. |
| Circles | **Understand and apply theorems about circles.** | **G.C.3 – Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.**<br>• Define inscribed and circumscribed circles of a triangle<br>• Recall midpoint and bisector definitions<br>• Define a point of concurrency<br>• Prove properties of angles for a quadrilateral inscribed in a circle<br>• Construct inscribed circles of a triangle<br>• Construct circumscribed circles of a triangle | 4. Model with mathematics.<br>5. Use appropriate tools strategically.<br>6. Attend to precision. |
| Circles | **Understand and apply theorems about circles.** | **G.C.4 – (+) Construct a tangent line from a point outside a given circle to the circle.**<br>• Define tangent, radius, perpendicular bisector, and midpoint<br>• Identify the center of the circle<br>• Synthesize the theory that applies to the circles that tangents drawn from a common external point are congruent<br>• Synthesize the theory that applies to circles that a radius is perpendicular to a tangent at the point of tangency<br>• Construct the perpendicular bisector of the line segment between the center, C to the outside point, P<br>• Construct arcs on circle C from the midpoint Q, having length of CQ<br>• Construct a tangent line | 5. Use appropriate tools strategically. |
| Circles | **Find are lengths and areas of sectors of circles.** Emphasize the similarity of all circles. Note that by similarity of sectors with the same central angle, arc lengths are proportional to the radius. Use this as a basis for introducing radian as a unit of measure. It is not intended that it be applied to the development of circular trigonometry in this course. | **G.C.5 – Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.**<br>• Recall how to find the area and circumference of a circle<br>• Explain that $1^\circ = \pi/180$ radians<br>• Determine the constant of proportionality (scale factor)<br>• Justify the radii of any two circles ($r1$ and $r2$) and the arc lengths ($s1$ and $s2$) determined by congruent central angles are proportional<br>• Verify that the constant of a proportion is the same as the radian measure, $\Theta$, of the given central angle. Conclude $s = r\Theta$ | 2. Reason abstractly and quantitatively.<br>3. Construct viable arguments and critique the reasoning of others.<br>5. Use appropriate tools strategically. |

| Geometry | | | |
|---|---|---|---|
| Unit 5: Circles with and without Coordinates | | | |
| Domains | Clusters with Instructional Notes | Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives | Mathematical Practices |
| Expressing Geometric Properties with Equations | **Translate between the geometric description and the equation for a conic section.** | **G.GPE.1 – Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.**<br>• **Define a circle**<br>• Use Pythagorean Theorem<br>• Complete the square of a quadratic equation<br>• Derive equation of a circle using the Pythagorean Theorem; given coordinates of the center and length of the radius<br>• Determine the center and radius by completing the square | 1. Make sense of problems and persevere in solving them.<br>2. Reason abstractly and quantitatively. |
| Expressing Geometric Properties with Equations | **Use coordinates to prove simple geometric theorems algebraically.** *Include simple proofs involving circles.* | **G.GPE.4 – Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, the square root of 3) lies on the circle centered at the origin and containing the point (0, 2).**<br>• Recall previous understandings of coordinate geometry including distance, midpoint, and slope formulas, equation of a line, and definitions of parallel and perpendicular lines<br>• Use coordinates to prove simple geometric theorems algebraically<br>• Derive the equation of a line through 2 points using similar right triangles<br>• Derive simple proofs involving circles | 1. Make sense of problems and persevere in solving them.<br>2. Reason abstractly and quantitatively.<br>3. Construct viable arguments and critique the reasoning of others.<br>6. Attend to precision. |
| Modeling with Geometry | **Apply geometric concepts in modeling situations.** *Focus on situations in which the analysis of circles in required.* | **G.MG.1 – Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).**<br>• Given a real world object, classify the object as a known geometric shape – use this to solve problems in context<br>• Focus on situations well modeled by trigonometric ratios for acute angles<br>• Use measure and properties of geometric shapes to describe real world objects | 4. Model with mathematics.<br>7. Look for and make sense of structure. |

| Geometry | | | |
|---|---|---|---|
| Unit 6: Applications of Probability | | | |
| Domains | Clusters with Instructional Notes | Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives | Mathematical Practices |
| Conditional Probability and the Rules of Probability | **Understand independence and conditional probability and use them to interpret data.** *Build on work with two-way tables from Algebra I Unit 3 (S.ID.5) to develop understanding of conditional probability and independence.* | **S.CP.1 – Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or", "and", "not").**<br>• Define unions, intersections and complements of events<br>• Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events | 2. Reason abstractly and quantitatively. |
| Conditional Probability and the Rules of Probability | **Understand independence and conditional probability and use them to interpret data.** *Build on work with two-way tables from Algebra I Unit 3 (S.ID.5) to develop understanding of conditional probability and independence.* | **S.CP.2 – Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.**<br>• Categorize events as independent or not using the characterization that two events A and B are independent when the probability of A and B occurring together is the product of their probabilities | 2. Reason abstractly and quantitatively. |
| Conditional Probability and the Rules of Probability | **Understand independence and conditional probability and use them to interpret data.** *Build on work with two-way tables from Algebra I Unit 3 (S.ID.5) to develop understanding of conditional probability and independence.* | **S.CP.3 – Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.**<br>• Know the conditional probability of A given B as P(A and B)/P(B)<br>• Interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B | 2. Reason abstractly and quantitatively. |
| Conditional Probability and the Rules of Probability | **Understand independence and conditional probability and use them to interpret data.** *Build on work with two-way tables from Algebra I Unit 3 (S.ID.5) to develop understanding of conditional probability and independence.* | **S.CP.4 – Construct and interpret two-way frequency tables of data including information from Montana American Indian data sources when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.**<br>• Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities<br>• Build on work with two way tables from Algebra 1 Unit 3 S-ID.5 to develop understanding of conditional probability and independence<br>• Interpret two-way frequency tables of data when two categories are associated with each object being classified | 3. Construct viable arguments and critique the reasoning of others. |
| Conditional Probability and the Rules of Probability | **Understand independence and conditional probability and use them to interpret data.** *Build on work with two-way tables from Algebra I Unit 3 (S.ID.5) to develop understanding of conditional probability and independence.* | **S.CP.5 – Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lunch cancer if you are a smoker with the chance of being a smoker if you have lung cancer.**<br>• Recognize the concepts of conditional probability and independence in everyday language and everyday situations<br>• Explain the concepts of conditional probability and independence in everyday language and everyday situations | 3. Construct viable arguments and critique the reasoning of others. |

| Geometry | | | |
|---|---|---|---|
| Unit 6: Applications of Probability | | | |
| Domains | Clusters with Instructional Notes | Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives | Mathematical Practices |
| Conditional Probability and the Rules of Probability | **Use rules of probability to compute probabilities of compound events in a uniform probability model.** | **S.CP.6 – Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.**<br>• Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A —<br>• Interpret the answer in terms of the model | 2. Reason abstractly and quantitatively. |
| Conditional Probability and the Rules of Probability | **Use rules of probability to compute probabilities of compound events in a uniform probability model.** | **S.CP.7 – Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.**<br>• Use the Additional Rule, P(A or B) = P(A) + P(B) – P(A and B)<br>• Interpret the answer in terms of the model | 3. Construct viable arguments and critique the reasoning of others. |
| Conditional Probability and the Rules of Probability | **Use rules of probability to compute probabilities of compound events in a uniform probability model.** | **S.CP.8 – (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B/A) = P(B)P(A/B), and interpret the answer in terms of the model.**<br>• Use the multiplication rule with correct notation<br>• Apply the general Multiplication Rule in a uniform probability model P(A and B) = P(A)P(B/A) = P(B)P(A/B)<br>• Interpret the answer in terms of the model | 2. Reason abstractly and quantitatively. |
| Conditional Probability and the Rules of Probability | **Use rules of probability to compute probabilities of compound events in a uniform probability model.** | **S.CP.9 – (+) Use permutations and combinations to compute probabilities of compound events and solve problems.**<br>• Identify situations that are permutations and those that are combinations<br>• Use permutations and combinations to compute probabilities of compound events and solve problems | 2. Reason abstractly and quantitatively. |
| Using Probability to Make Decisions | **Use probability to evaluate outcomes of decisions.** *This unit sets the stage for work in Algebra II, where the idea of statistical inference are introduced. Evaluating the risks associated with conclusions drawn from sample data (i.e. incomplete information) requires an understanding of probability concepts.* | **S.MD.6 – (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).**<br>• Compute Theoretical and Experimental Probabilities<br>• Recall previous understandings of probability<br>• Use probabilities to make fair decisions<br>• Extend to more complex probability models. Include situations such as those involving quality control, or diagnostic tests that yield both false positive and false negative results | 2. Reason abstractly and quantitatively.<br>3. Construct viable arguments and critique the reasoning of others.<br>4. Model with mathematics.<br>5. Use appropriate tools strategically.<br>8. Look for and express regularity in repeated reasoning. |

| Geometry | | | |
|---|---|---|---|
| Unit 6: Applications of Probability | | | |
| Domains | Clusters with Instructional Notes | Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives | Mathematical Practices |
| Using Probability to Make Decisions | **Use probability to evaluate outcomes of decisions.** *This unit sets the stage for work in Algebra II, where the idea of statistical inference are introduced. Evaluating the risks associated with conclusions drawn from sample data (i.e. incomplete information) requires an understanding of probability concepts.* | **S.MD.7 – (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).**<br>• Recall previous understandings of probability<br>• Analyze decisions and strategies using probability concepts<br>• Extend to more complex probability models. Include situations such as those involving quality control, or diagnostic tests that yield both false positive and false negative results | 1. Make sense of problems and persevere in solving them.<br>2. Reason abstractly and quantitatively.<br>3. Construct viable arguments and critique the reasoning of others.<br>6. Attend to precision. |

# GLOSSARY

**Addition and subtraction within 5, 10, 20, 100, or 1000.** Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: $8 + 2 = 10$ is an addition within 10, $14 - 5 = 9$ is a subtraction within 20, and $55 - 18 = 37$ is a subtraction within 100.

**Additive inverses.** Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because $3/4 + (-3/4) = (-3/4) + 3/4 = 0$.

**Associative property of addition.** See Table 3 in this Glossary.

**Associative property of multiplication.** See Table 3 in this Glossary.

**Bivariate data.** Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

**Box plot.** A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1]

**Commutative property.** See Table 3 in this Glossary.

**Complex fraction.** A fraction $A/B$ where $A$ and/or $B$ are fractions ($B$ nonzero).

**Computation algorithm.** A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

**Computation strategy.** Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

**Congruent.** Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

**Counting on.** A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by *counting on*—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

**Dot plot.** *See:* line plot.

**Dilation.** A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

**Expanded form.** A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, $643 = 600 + 40 + 3$.

**Expected value.** For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

**First quartile.** For a data set with median $M$, the first quartile is the median of the data values less than $M$. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[2] *See also:* median, third quartile, interquartile range.

[1] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.
[2] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," *Journal of Statistics Education* Volume 14, Number 3 (2006).

**Fraction.** A number expressible in the form *a*/*b* where *a* is a whole number and *b* is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.) *See also:* rational number.

**Identity property of 0.** See Table 3 in this Glossary.

**Independently combined probability models.** Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

**Integer.** A number expressible in the form $a$ or $-a$ for some whole number $a$.

**Interquartile Range.** A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also:* first quartile, third quartile.

**Line plot.** A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

**Mean.** A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.4 Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

**Mean absolute deviation.** A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

**Median.** A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

**Midline.** In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

**Multiplication and division within 100.** Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

**Multiplicative inverses.** Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 × 4/3 = 4/3 × 3/4 = 1.

**Number line diagram.** A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

**Percent rate of change.** A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

**Probability distribution.** The set of possible values of a random variable with a probability assigned to each.

**Properties of operations.** See Table 3 in this Glossary.

**Properties of equality.** See Table 4 in this Glossary.

[3] Adapted from Wisconsin Department of Public Instruction, *op. cit.*
[4] To be more precise, this defines the *arithmetic mean.* one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

**Properties of inequality**. See Table 5 in this Glossary.

**Properties of operations**. See Table 3 in this Glossary.

**Probability**. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

**Probability model.** A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

**Random variable.** An assignment of a numerical value to each outcome in a sample space.

**Rational expression.** A quotient of two polynomials with a non-zero denominator.

**Rational number**. A number expressible in the form $a/b$ or $-a/b$ for some fraction $a/b$. The rational numbers include the integers.

**Rectilinear figure.** A polygon all angles of which are right angles.

**Rigid motion.** A transformation of points in space consisting of a sequence of

**Repeating decimal**. The decimal form of a rational number. *See also:* terminating decimal.

**Sample space**. In a probability model for a random process, a list of the individual outcomes that are to be considered.

**Scatter plot**. A graph in the coordinate plane representing a set of Bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[5]

**Similarity transformation**. A rigid motion followed by a dilation.

**Tape diagram**. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

**Terminating decimal.** A decimal is called terminating if its repeating digit is 0.

**Third quartile**. For a data set with median $M$, the third quartile is the median of the data values greater than $M$. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.

**Transitivity principle for indirect measurement.** If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

**Uniform probability model**. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

**Vector.** A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

**Visual fraction model.** A tape diagram, number line diagram, or area model.

**Whole numbers**. The numbers 0, 1, 2, 3, . . .

[5]Adapted from Wisconsin Department of Public Instruction, *op. cit.*

# Tables

## Table 1. Common addition and subtraction situations.[1]

| | Result Unknown | Change Unknown | Start Unknown |
|---|---|---|---|
| **Add to** | Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now?<br>$2 + 3 = ?$ | Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$ | Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before?<br>$? + 3 = 5$ |
| **Take from** | Five apples were on the table. I ate two apples. How many apples are on the table now?<br>$5 - 2 = ?$ | Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?<br>$5 - ? = 3$ | Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?<br>$? - 2 = 3$ |

| | Total Unknown | Addend Unknown | Both Addends Unknown[2] |
|---|---|---|---|
| **Put Together/ Take Apart[1]** | Three red apples and two green apples are on the table. How many apples are on the table?<br>$3 + 2 = ?$ | Five apples are on the table. Three are red and the rest are green. How many apples are green?<br>$3 + ? = 5, 5 - 3 = ?$ | Grandma has five flowers. How many can she put in her red vase and how many in her blue vase?<br>$5 = 0 + 5, 5 = 5 + 0$<br>$5 = 1 + 4, 5 = 4 + 1$<br>$5 = 2 + 3, 5 = 3 + 2$ |

| | Difference Unknown | Bigger Unknown | Smaller Unknown |
|---|---|---|---|
| **Compare[2]** | ("How many more?" version):<br>Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?<br><br>("How many fewer?" version):<br>Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5, 5 - 2 = ?$ | (Version with "more"):<br>Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?<br><br>(Version with "fewer"):<br>Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have?<br>$2 + 3 = ?, 3 + 2 = ?$ | (Version with "more"):<br>Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?<br><br>(Version with "fewer"):<br>Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have?<br>$5 - 3 = ?, ? + 3 = 5$ |

[1]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

[1]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

**Table 2. Common multiplication and division situations.**[1]

| | Unknown Product<br>$3 \times 6 = ?$ | Group Size Unknown<br>("How many in each group?" Division)<br>$3 \times ? = 18$, and $18 \div 3 = ?$ | Number of Groups Unknown<br>("How many groups?" Division)<br>$? \times 6 = 18$, and $18 \div 6 = ?$ |
|---|---|---|---|
| **Equal Groups** | There are 3 bags with 6 plums in each bag. How many plums are there in all?<br><br>*Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether? | If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?<br><br>*Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be? | If 18 plums are to be packed 6 to a bag, then how many bags are needed?<br><br>*Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have? |
| **Arrays,[4] Area[5]** | There are 3 rows of apples with 6 apples in each row. How many apples are there?<br><br>*Area example.* What is the area of a 3 cm by 6 cm rectangle? | If 18 apples are arranged into 3 equal rows, how many apples will be in each row?<br><br>*Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it? | If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?<br><br>*Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it? |
| **Compare** | A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?<br><br>*Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long? | A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?<br><br>*Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first? | A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat?<br><br>*Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first? |
| **General** | $a \times b = ?$ | $a \times ? = p$, and $p \div a = ?$ | $? \times b = p$, and $p \div b = ?$ |

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[1]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

**Table 3. The properties of operations.** **Here $a$, $b$ and $c$ stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.**

| | |
|---|---|
| *Associative property of addition* | $(a + b) + c = a + (b + c)$ |
| *Commutative property of addition* | $a + b = b + a$ |
| *Additive identity property of 0* | $a + 0 = 0 + a = a$ |
| *Existence of additive inverses* | For every $a$ there exists $-a$ so that $a + (-a) = (-a) + a = 0$ |
| *Associative property of multiplication* | $(a \times b) \times c = a \times (b \times c)$ |
| *Commutative property of multiplication* | $a \times b = b \times a$ |
| *Multiplicative identity property of 1* | $a \times 1 = 1 \times a = a$ |
| *Existence of multiplicative inverses* | For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$ |
| *Distributive property of multiplication over addition* | $a \times (b + c) = a \times b + a \times c$ |

**Table 4. The properties of equality.** **Here $a$, $b$ and $c$ stand for arbitrary numbers in the rational, real, or complex number systems.**

| | |
|---|---|
| *Reflexive property of equality* | $a = a$ |
| *Symmetric property of equality* | If $a = b$, then $b = a$ |
| *Transitive property of equality* | If $a = b$ and $b = c$, then $a = c$ |
| *Addition property of equality* | If $a = b$, then $a + c = b + c$ |
| *Subtraction property of equality* | If $a = b$, then $a - c = b - c$ |
| *Multiplication property of equality* | If $a = b$, then $a \times c = b \times c$ |
| *Division property of equality* | If $a = b$ and $c \neq 0$, then $a \div c = b \div c$ |
| *Substitution property of equality* | If $a = b$, then $b$ may be substituted for $a$ in any expression containing $a$. |

**Table 5. The properties of inequality.** **Here $a$, $b$ and $c$ stand for arbitrary numbers in the rational or real number systems.**

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.

If $a > b$ and $b > c$ then $a > c$.

If $a > b$, then $b < a$.

If $a > b$, then $-a < -b$.

If $a > b$, then $a \pm c > b \pm c$.

If $a > b$ and $c > 0$, then $a \times c > b \times c$.

If $a > b$ and $c < 0$, then $a \times c < b \times c$.

If $a > b$ and $c > 0$, then $a \div c > b \div c$.

If $a > b$ and $c < 0$, then $a \div c < b \div c$.

## LEARNING PROGRESSIONS BY DOMAIN

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