Billings
PUBLIC SCHOOLS

Mathematical Practice and Content

Common Core Standards

Algebra I

March 2012

PHILOSOPHY

We believe every student can understand the general nature and uses of mathematics necessary to solve problems, reason inductively and deductively and apply numerical concepts necessary to function in a technological society. We believe instructional strategies must include real world applications and the appropriate use of technology. We believe students must be able to use mathematics as a communications medium.

Therefore, as an educational system we believe we can teach all children and all children can learn. We believe accessing knowledge, reasoning, questioning, and problem solving are the foundations for learning in an ever-changing world. We believe education enables students to recognize and strive for higher standards. Consequently, we will commit out efforts to help students acquire knowledge and attitudes considered valuable in order to develop their potential and/or their career and lifetime aspirations.

MATHEMATICAL PRACTICES

The Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12.

1. Make sense of problems and persevere in solving them.

Mathematically proficient students:

- **a. Understand that mathematics is relevant when studied in a cultural context.**
- **b. Explain the meaning of a problem and restate it in their words.**
- **c. Analyze given information to develop possible strategies for solving the problem.**
- **d. Identify and execute appropriate strategies to solve the problem.**
- **e. Evaluate progress toward the solution and make revisions if necessary.**
- **f. Check their answers using a different method, and continually ask "Does this make sense?"**

2. Reason abstractly and quantitatively.

Mathematically proficient students:

- **a. Make sense of quantities and their relationships in problem situations.**
- **b. Use varied representations and approaches when solving problems.**
- **c. Know and flexibly use different properties of operations and objects.**
- **d. Change perspectives, generate alternatives and consider different options.**

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students:

- **a. Understand and use prior learning in constructing arguments.**
- **b. Habitually ask "why" and seek an answer to that question.**
- **c. Question and problem-pose.**
- **d. Develop questioning strategies to generate information.**
- **e. Seek to understand alternative approaches suggested by others and. As a result, to adopt better approaches.**

f. Justify their conclusions, communicate them to others, and respond to the arguments of others.
g. Compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.

4. Model with mathematics.
Mathematically proficient students:
a. Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. This includes solving problems within a cultural context, including those of Montana American Indians.
b. Make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.
c. Identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
d. Analyze mathematical relationships to draw conclusions.

5. Use appropriate tools strategically.
Mathematically proficient students:
a. Use tools when solving a mathematical problem and to deepen their understanding of concepts (e.g., pencil and paper, physical models, geometric construction and measurement devices, graph paper, calculators, computer-based algebra or geometry systems.)
b. Make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. They detect possible errors by strategically using estimation and other mathematical knowledge.

6. Attend to precision.
Mathematically proficient students:
a. Communicate their understanding of mathematics to others.
b. Use clear definitions and state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
c. Specify units of measure and use label parts of graphs and charts
d. Strive for accuracy.

7. Look for and make use of structure.
Mathematically proficient students:
a. Look for, develop, generalize and describe a pattern orally, symbolically, graphically and in written form.
b. Apply and discuss properties.

8. Look for and express regularity in repeated reasoning.
Mathematically proficient students:
a. Look for mathematically sound shortcuts.
b. Use repeated applications to generalize properties.

Traditional Pathway: High School Algebra 1

The fundamental purpose of this course is to formalize and extend the mathematics that students learned in the middle grades. Since it is built on the middle grades standards, this is a more ambitious version of Algebra 1 than has generally been offered. The critical areas, called units, deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. The Mathematical Practice Standards apply throughout each course and, together, with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Critical Area/Unit 1: By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

Critical Area/Unit 2: In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notations and develop the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. Students explore systems of equations and inequalities, and they find and interpret their solutions. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Critical Area/Unit 3: This unit builds upon students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Critical Area/Unit 4: In this unit, students build on their knowledge from unit 2, where they extended the laws of exponents to rational exponents. Students apply this new understanding of number and strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions.

Critical Area/Unit 5: In this unit, students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students expand their experience with functions to include more specialized functions-absolute value, step, and those that are piecewise-defined.

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~Interpret the structure of expressions
~Create equations that describe numbers or relationships
~Understand solving equations as a process of reasoning and explain the reasoning
~Solve equations and inequalities in one variable</td><td rowspan="5">1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.</td></tr>
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Linear and Exponential Relationships</td><td>~Extend the properties of exponents to rational exponents
~Solve systems of equations
~Represent and solve equations and inequalities graphically
~Understand the concept of a function and use function notation
~Interpret functions that arise in applications in terms of a context
~Analyze functions using different representations
~Build a function that models a relationship between two quantities
~Build new functions from existing functions
~Construct and compare linear, quadratic, and exponential models and solve problems
~Interpret expressions for functions in terms of the situation they model</td></tr>
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Expressions and Equations</td><td>~Interpret the structure of expressions
~Write expressions in equivalent forms to solve problems
~Perform arithmetic operations on polynomials
~Create equations that describe numbers or relationships
~Solve equations and inequalities in one variable
~Solve systems of equations</td></tr>
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~Interpret functions that arise in applications in terms of a context
~Analyze functions using different representations
~Build a function that models a relationship between two quantities
~Build new functions from existing functions
~Construct and compare linear, quadratic, and exponential models and solve problems</td></tr>
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Algebra 1			
Unit 1: Relationships between Quantities and Reasoning with Equations			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
	Skills to maintain-*Reinforce understanding of the properties of integer exponents. The initial experience with exponential expressions, equations, and functions involves integer exponents and builds on this understanding.*		
Quantities	**Reason quantitatively and use units to solve problems.** *Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.*	**N.Q.1 – Use units as a way to understand problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians, and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.** • Recognize units given or needed to solve problem • Use given units and the context of a problem as a way to determine if the solution to a multi-step problem is reasonable • Choose appropriate units to represent a problem when using formulas or graphing • Interpret units or scales used in formulas or represented in graphs • Use units as a way to understand problems and to guide the solution of multi-step problems • Calculate unit conversions	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 4. Model with mathematics. 6. Attend to precision.
Quantities	**Reason quantitatively and use units to solve problems.** *Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.*	**N.Q.2 – Define appropriate quantities for the purpose of descriptive modeling.** • Define descriptive modeling • Determine appropriate quantities for the purpose of descriptive modeling	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 4. Model with mathematics. 6. Attend to precision 8. Look for and express regularity is repeated reasoning.
Quantities	**Reason quantitatively and use units to solve problems.** *Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.*	**N.Q.3 – Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.** • Identify appropriate units of measurement to report quantities • Determine the limitations of different measurement tools • Choose and justify a level of accuracy and/or precision appropriate to limitations on measurement when reporting quantities • Identify important quantities in a problem or real-world context	2. Reason abstractly and quantitatively. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision.

Algebra 1			
Unit 1: Relationships between Quantities and Reasoning with Equations			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Seeing Structure in Expressions	**Interpret the structure of expressions.** *Limit to linear expressions and to exponential expressions with integer exponents.*	**A.SSE.1 – Interpret expressions that represent a quantity in terms of its context** (Modeling standard). **a. Interpret parts of an expression, such as terms, factors, and coefficients.** **b. Interpret complicated expressions by viewing one or more of their parts as a single entity, For example, interpret P(1+r)n as the product of P and a factor not depending on P.** • Define and recognize parts of an expression, such as terms, factors, and coefficients • Interpret parts of an expression, such as terms, factors, and coefficients in terms of the context • Interpret complicated expressions, in terms of the context, by viewing one or more of their parts as a single entity	2. Reason abstractly and quantitatively. 4. Model with mathematics. 7. Look for and make use of structure.

Algebra 1			
Unit 1: Relationships between Quantities and Reasoning with Equations			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Creating Equations	**Create equations that describe numbers or relationships.** *Limit A.CED.1 and A.CED.2 to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs. Limit A.CED.3 to linear equations and inequalities. Limit A.CED.4 to formulas which are linear in the variable of interest.*	**A.CED.1 – Create equations and inequalities in one variable and use them to solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.** • Describe the relationships between the quantities in the problem (for example, how the quantities are changing or growing with respect to each other); express these relationships using mathematical operations to create an appropriate equation or inequality to solve • Use all available types of functions to create such equations, including root functions, but constrain to simple cases • Compare and contrast problems that can be solved by different types of equations • Compare and contrast problems that can be solved by different types of equations (linear, exponential) • Solve linear and exponential equations in one variable • Solve inequalities in one variable • Solve all available types of equations and inequalities including root equations and inequalities, in one variable • Create equations and inequalities in one variable and use them to solve problems • Create equations and inequalities in one variable to model real-world situations • Create equations (linear, exponential) and inequalities in one variable and use them to solve problems	1. Make sense of problems and persevere in solving them. 4. Model with mathematics. 8. Look for and express regularity in repeated reasoning.

Algebra 1			
Unit 1: Relationships between Quantities and Reasoning with Equations			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Creating Equations	**Create equations that describe numbers or relationships.** *Limit A.CED.1 and A.CED.2 to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs. Limit A.CED.3 to linear equations and inequalities. Limit A.CED.4 to formulas which are linear in the variable of interest.*	**A.CED.2 – Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.** • Identify the quantities in a mathematical problem or real-world situation that should be represented by distinct variables and describe what quantities the variable represent • Graph one or more created equation on coordinate axes with appropriate labels and scales • Justify which quantities in a mathematical problem or real-world situation are dependent and independent of one another and which operations represent those relationships • Determine appropriate units for the labels and scale of graph depicting the relationship between equations created in two or more variables • Create at least two equations in two or more variables to represent relationships between quantities	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision.

Algebra 1			
Unit 1: Relationships between Quantities and Reasoning with Equations			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Creating Equations	**Create equations that describe numbers or relationships.** *Limit A.CED.1 and A.CED.2 to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs. Limit A.CED.3 to linear equations and inequalities. Limit A.CED.4 to formulas which are linear in the variable of interest.*	**A.CED.3 – Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.** • Recognize when a modeling context involves constraints • Interpret solutions as viable or nonviable options in a modeling context • Determine when a problem should be represented by equations, inequalities, systems of equations and/or inequalities • Represent constraints by equations or inequalities, and by systems of equations and/or inequalities	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 4. Model with mathematics. 5. Use appropriate tools strategically.
Creating Equations	**Create equations that describe numbers or relationships.** *Limit A.CED.1 and A.CED.2 to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs. Limit A.CED.3 to linear equations and inequalities. Limit A.CED.4 to formulas which are linear in the variable of interest.*	**A.CED.4 – Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.** • Define a quantity of interest to mean any numerical or algebraic quantity (e.g., 2(a/b)=d in which 2 is the quantity of interest showing that d must be even; (ϖr2h/3)=Vcone and ϖr2h=Vcylinder showing that Vcylinder=3*Vcone) • Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. (e.g., ϖr2 can be re written as (ϖr)r which makes the form of this expression resemble Bh. The quantity of interest could also be (a + b)n = an b0 + a(n-1)b1 +...+ a0b n)	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 4. Model with mathematics. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Algebra 1			
Unit 1: Relationships between Quantities and Reasoning with Equations			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Reasoning with Equations and Inequalities	**Understand solving equations as a process of reasoning and explain the reasoning.** *Students should focus on and master A.REI.1 for linear equations and be able to extend and apply their reasoning to other types of equations in future courses. Students will solve exponential equations with logarithms in Algebra II*	**A.REI.1 – Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.** • Know that solving an equation means that the equation remains balanced during each step • Recall the properties of equality • Explain why, when solving equations, it is assumed that the original equation is equal • Determine if an equation has a solution • Choose an appropriate method for solving the equation • Justify solutions(s) to equations by explaining each step in solving a simple equation using the properties of equality, beginning with the assumption that the original equation is equal • Construct a mathematically viable argument justifying a given, or self-generated, solution method	1. Make sense of problems and persevere in solving them. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Reasoning with Equations and Inequalities	**Solve equations and inequalities in one variable.** *Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents.*	**A.REI.3 – Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.** • Recall properties of equality • Determine the effect that rational coefficients have on the inequality symbol and use this to find the solution set • Solve multi-step equations in one variable • Solve multi-step inequalities in one variable • Solve equations and inequalities with coefficients represented by letters	1. Make sense of problems and persevere in solving them. 6. Attend to precision. 7. Look for and make use of structure.

Algebra 1			
Unit 2: Linear and Exponential Relationships			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
The Real Number System	**Extend the properties of exponents to rational exponents.** *In implementing the standards in curriculum, these standards should occur before discussing exponential functions with continuous domains.*	**N.RN.1 – Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define 5 1/3 to be the cube root of 5 because we want (5 1/3)3 = 5(1/3)3 to hold, so (5 1/3)3 must equal 5.** • Define radical notation as a convention used to represent rational exponents • Explain the properties of operations of rational exponents as an extension of the properties of integer exponents • Explain how radical notation, rational exponents, and properties of integer exponents relate to one another	2. Reason abstractly and quantitatively. 7. Look for and make use of structure.
The Real Number System	**Extend the properties of exponents to rational exponents.** *In implementing the standards in curriculum, these standards should occur before discussing exponential functions with continuous domains.*	**N.RN.2 – Rewrite expressions involving radicals and rational exponents using the properties of exponents.** • Using the properties of exponents, rewrite a radical expression as an expression with a rational exponent • Using the properties of exponents, rewrite an expression with a rational exponent as a radical expression	7. Look for and make use of structure.
Reasoning with Equations and Inequalities	**Solve systems of equations.** *Build on student experiences graphing and solving systems of linear equations from middle school to focus on justification of the methods used. Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution);connect to GPE.5 when it is taught in Geometry, which required students to prove the slope criteria for parallel lines.*	**A.REI.5 – Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.** • Recognize and use properties of equality to maintain equivalent systems of equations • Justify that replacing one equation in a two-equation system with the sum of that equation and a multiple of the other will yield the same solutions as the original system	1. Make sense of problems and persevere in solving them. 3. Construct viable arguments and critique the reasoning of others. 6. Attend to precision. 8. Look for and express regularity in repeated reasoning.
Reasoning with Equations and Inequalities	**Represent and solve equations and inequalities graphically.** *For A.REI.10, focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.*	**A.REI.10 – Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).** • Recognize that the graphical representation of an equation in two variables is a curve, which may be a straight line • Explain why each point on a curve is a solution to its equation	7. Look for an make use of structure.

Algebra 1			
Unit 2: Linear and Exponential Relationships			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Reasoning with Equations and Inequalities	**Represent and solve equations and inequalities graphically.** *For A.REI.11, focus on cases where f(x) and g(x) are linear or exponential.*	**A.REI.11 – Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.** • Recognize and use function notation to represent linear and exponential equations • Recognize that if (x1, y1) and (x2, y2) share the same location in the coordinate plane that x1 = x2 and y1 = y2 • Recognize that f(x) = g(x) means that there may be particular inputs of f and g for which the outputs of f and g are equal • Recognize and use function notation to represent linear, polynomial, rational, absolute value, exponential, and radical equations • Explain why the x-coordinates of the points where the graph of the equations y = f(x) and y = g(x) intersect are the solutions of the equations f(x) = g(x) • Approximate/find the solutions(s) using an appropriate method. For example, using technology to graph the functions, make tables of values or find successive approximations	3. Construct variable arguments and critique the reasoning of other. 4. Model with Mathematics. 5. Use appropriate tools strategically. 6. Attend to precision.
Reasoning with Equations and Inequalities	**Represent and solve equations and inequalities graphically.**	**A.REI.12 – Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.** • Identify characteristics of a linear inequality and a system of linear inequalities, such as; boundary line and shading, and determine the appropriate points to test and derive a solution set from • Explain the meaning of the intersection of shaded regions in a system of linear inequalities • Graph a line or boundary line and shade the appropriate region from a two-variable linear inequality† • Graph a system of linear inequalities and shade the appropriate overlapping region for a system of linear inequalities	5. Use appropriate tools strategically. 6. Attend to precision.
Interpreting Functions	**Understand the concept of a function and use function notation.** *Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of functions at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear and exponential functions.*	**F.IF.1 – Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, the f(x) denotes the output of f corresponding to the input x The graph of f is the graph of the equation y = f(x).** • Identify the domain and range of a function • Determine if a relation is a function • Determine the value of the function with proper notation • Evaluate functions for given values of x	2. Reason abstractly and quantitatively. 4. Model with mathematics. 7. Look for and make use of structure.

Algebra 1			
Unit 2: Linear and Exponential Relationships			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Interpreting Functions	**Understand the concept of a function and use function notation.** *Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of functions at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear and exponential functions.*	**F.IF.2 - Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.** • Identify mathematical relationships and express them using function notation • Define a reasonable domain, which depends on the context and/or mathematical situation, for a function focusing on linear and exponential functions • Evaluate functions at a given input in the domain, focusing on linear and exponential functions • Interpret statements that use functions in terms of real world situations, focusing on linear and exponential functions	1. Make sense of problems and persevere. 2. Reason abstractly and quantitatively. 4. Model with mathematics. 5. Use appropriate tools strategically. 7. Look for and make use of structure.
Interpreting Functions	**Understand the concept of a function and use function notation.** *Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of functions at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear and exponential functions. In F.IF.3, draw conncetion to F.BF.2, which requires students to write arithmetic and geometric sequences. Emphasize arithmetic and geometric sequences as examples of linear and exponential functions.*	**F.IF.3 – Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ for $n \geq 1$.** • Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of integers	4. Model with mathematics. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Algebra 1			
Unit 2: Linear and Exponential Relationships			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Interpreting Functions	**Interpret functions that arise in applications in terms of a context.** *For F.IF.4 and 5, focus on linear and exponential functions.*	**F.IF.4 – For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.** • Define and recognize key features in tables and graphs of linear and exponential functions; intercepts; intervals where the function is increasing, decreasing, positive, or negative, and end behavior • Define and recognize key features in tables and graphs of linear, exponential, and quadratic functions: intercepts; intervals where the function is increasing, decreasing, positive, or negative, relative maximums, symmetries, end behavior and periodicity • Identify the type of function, given a table or graph • Identify whether a function is linear or exponential, given its table or graph • Interpret key features of graphs and tables of functions in terms of the contextual quantities each function represents • Sketch graphs showing the key features of a function, modeling a relationship between two quantities, given a verbal description of the relationship	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 4. Model with mathematics.
Interpreting Functions	**Interpret functions that arise in applications in terms of a context.** *For F.IF.4 and 5, focus on linear and exponential functions.*	**F.IF.5 – Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.** • Identify and describe the domain of a function, given the graph or a verbal/written description of a function • Identify an appropriate domain based on the unit, quantity, and type of function it describes • Relate the domain of a function to its graph and to the quantitative relationship it describes, where applicable • Explain why a domain is appropriate for a given situation	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision.
Interpreting Functions	**Interpret functions that arise in applications in terms of a context.** *For F.IF.6, focus on linear functions and exponential functions whose domain is a subset of the integers. Unit 5 in this course and the Algebra II course address other types of functions.*	**F.IF.6 – Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.** • Recognize slope as an average rate of change • Estimate the rate of change from a linear or exponential graph • Interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval • Calculate the average rate of change of a function (presented symbolically or as a table) over a specified interval	2. Reason abstractly and quantitatively. 4. Model with mathematics. 5. Use appropriate tools strategically.

Algebra 1			
Unit 2: Linear and Exponential Relationships			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Interpreting Functions	**Analyze functions using different representations.** *For F.IF.7a, 7e, and 9 focus on linear and exponentials functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as y=3 to the n power and y=100 squared*	**F.IF.7 – Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.** **a. Graph linear and quadratic functions and show intercepts, maxima, and minimum** **e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.** • Determine the difference between simple and complicated polynomial functions • Determine the difference between simple and complicated linear, quadratic, square root, cube root, and piecewise-defined functions • Determine the differences between simple and complicated linear and exponential functions and know when the use of technology is appropriate • Compare and contrast absolute value, step-and piecewise-defined functions with linear, quadratic, and exponential functions • Compare and contrast the domain and range of absolute value, step-and piecewise-defined functions with linear, quadratic, and exponential functions • Compare and contrast the domain and range of exponential, logarithmic, and trigonometric functions with linear, quadratic, absolute value, step-and piecewise-defined functions • Analyze the difference between simple and complicated linear, quadratic, square root, cube root, piecewise-defined, exponential, logarithmic, and trigonometric functions, including step and absolute value functions • Select the appropriate type of function, taking into consideration the key features, domain, and range, to model a real-world situation • Relate the relationship between zeros of quadratic functions and their factored forms to the relationship between polynomial functions of degrees greater than two • Graph exponential functions, by hand in simple cases or using technology for more complicated cases, and show intercepts and end behavior • Graph polynomial functions, by hand in simple cases or using technology for more complicated cases, and show/label maxima and minima of the graph, identify zeros when suitable factorizations are available, and show end behavior • Graph exponential, logarithmic, and trigonometric functions, by hand in simple cases or using technology for more complicated cases. For exponential and logarithmic functions, show: period, midline, and amplitude • Graph linear functions by hand in simple cases or using technology for more complicated cases and show/label intercepts of the graph • Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions, by hand in simple case or using technology for more complicated cases, and show/label key features of the graph	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision.
Interpreting Functions	**Analyze functions using different representations.** *For F.IF.7a, 7e, and 9 focus on linear and exponentials functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as y=3 to the n power and y=100 squared*	**F.IF.9 – Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.** • Identify types of functions based on verbal, numerical, algebraic, and graphical descriptions and state key properties • Differentiate between exponential and linear functions using a variety of descriptors (graphical, verbal, numerical, algebraic) • Differentiate between different types of functions using a variety of descriptors (graphical, verbal, numerical, algebraic) • Use a variety of function representations (algebraic, graphical, numerical in tables, or by verbal descriptions) to compare and contrast properties of two functions	1. Make sense of problems and persevere in solving them. 5. Use appropriate tools strategically. 7. Look for and make use of structure.

Algebra 1			
Unit 2: Linear and Exponential Relationships			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Building Functions	**Build a function that models a relationship between two quantities.** *Limit F.BF.1a and1b, and 2 to linear and exponential functions.*	**F.BF.1 – Write a function that describes a relationship between two quantities.** **a. Determine an explicit expression, a recursive process, or steps for calculation from a context.** **b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.** • Define explicit function and recursive process • Combine two functions using the operations of addition, subtraction, multiplication, and division • Evaluate the domain of the combined function • Given a real-world situation or mathematical problem, build standard functions to represent relevant relationships/quantities • Given a real-world situation or mathematical problem, determine which arithmetic operation should be performed to build the appropriate combined function • Given a real-world situation or mathematical problem, relate the combined function to the context of the problem • Write a function that describes a relationship between two quantities by determining an explicit expression, a recursive process, or steps for calculation from a context	2. Reason abstractly and quantitatively. 4. Model with mathematics. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Building Functions	**Build a function that models a relationship between two quantities.** *Limit F.BF.1a and1b, and 2 to linear and exponential functions. In F.BF.2, connect arithmetic sequences to linear functions and geometric sequences to exponential functions.*	**F.BF.2 – Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations from a variety of contexts (e.g., science, history, and culture, including those of the Montana American Indian), and translate between the two forms.** • Identify arithmetic and geometric patterns in given sequences • Determine the recursive rule given arithmetic and geometric sequences • Determine the explicit formula given arithmetic and geometric sequences • Justify the translation between the recursive form and explicit formula for arithmetic and geometric sequences • Generate arithmetic and geometric sequences from recursive and explicit formulas • Given an arithmetic or geometric sequence in recursive form, translate into the explicit formula • Given an arithmetic or geometric sequence as an explicit formula, translate into the recursive form • Use given and construct arithmetic and geometric sequences, expressed both recursively and with explicit formulas, to model real-life situations	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Algebra 1			
Unit 2: Linear and Exponential Relationships			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Building Functions	**Build new functions from existing functions.** *Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept. While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.*	F.BF.3– Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. • Given a single transformation on a function symbolic or graphic identify the effect on the graph • Using technology, identify effects of single transformations on graphs of functions • Recognize even and odd functions from their graphs and equations • Describe the differences and similarities between a parent function and the transformed function • Find the value of k, given the graphs of a parent function, f(x), and the transformed function; f(x) + k, k f(x), f(kx), or f(x + k) • Experiment with cases and illustrate an explanation of the effects on a graph, using technology • Graph a given function by replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative)	4. Model with mathematics. 5. Use appropriate tools strategically. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Linear, Quadratic, and Exponential Models	**Construct and compare linear, quadratic, and exponential models and solve problems.**	F.LE.1 – Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. • Recognize that linear functions grow by equal differences over equal intervals • Recognize that exponential functions grow by equal factors over equal intervals • Recognize situations in which one quantity changes at a constant rate per unit interval (equal differences), relative to another to solve mathematical and real-world problems • Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval (equal factors), relative to another to solve mathematical and real-world problems • Distinguish between situations that can be modeled with linear functions and exponential functions to solve mathematical and real-world problems • Prove that linear functions grow by equal differences over equal intervals • Prove that exponential functions grow by equal factors over equal intervals	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Algebra 1			
Unit 2: Linear and Exponential Relationships			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Linear, Quadratic, and Exponential Models	**Construct and compare linear, quadratic, and exponential models and solve problems.** *In F.LE.2, draw on and consolidate previous work in Grade 8 on finding equations for lines and linear functions (8.EE.6, 8.F.4).*	F.LE.2 – Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). • Recognize that arithmetic sequences can be expressed as linear functions • Recognize that geometric sequences can be expressed as exponential functions • Determine when a graph, a description of a relationship, or two input-output pairs (include reading these from a table) represent a linear or exponential function in order so solve problems • Construct linear functions, including arithmetic sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table) • Construct exponential functions, including geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table)	1. Make sense of problems and persevere in solving them. 4. Model with mathematics. 5. Use appropriate tools strategically. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Linear, Quadratic, and Exponential Models	**Construct and compare linear, quadratic, and exponential models and solve problems.** *For F.LE.3, limit to comparisons between linear and exponential models.*	F.LE.3 – Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. • Informally define the concept of end behavior • Fluently compute growth rates for linear, exponential, and quadratic functions • Compare tables and graphs of linear and exponential functions to observe that a quantity, increasing exponentially, exceeds all others to solve mathematical and real-world problems • Compare tables and graphs of exponential and other polynomial functions to observe that a quantity, increasing exponentially, exceeds all others to solve mathematical and real-world problems	2. Reason abstractly and quantitatively. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 8. Look for and express regularity in repeated reasoning.
Linear, Quadratic, and Exponential Models	**Interpret expressions for functions in terms of the situation they model.** *Limit exponential functions to those of the form f(x)=b to the power of x +k.*	F.LE.5– Interpret the parameters in a linear or exponential function in terms of a context. • Recognize linear or exponential function including; vertical and horizontal shifts, vertical and horizontal dilations • Recognize rates of change and intercepts as parameters in linear or exponential functions • Interpret the parameters in a linear or exponential function in terms of a context	2. Reason abstractly and quantitatively. 4. Model with mathematics. 5. Use appropriate tools strategically. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Algebra 1			
Unit 3: Descriptive Statistics			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Interpreting Categorical and Quantitative Data	**Summarize, represent, and interpret data on a single count or measurement variable.** *In grades 6-8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution or the existence of extreme data points.*	**S.ID.1 – Represent data with plots on the real number line (dot plots, histograms, and box plots).** • Represent data with plots on the real number line, using various display types by creating dot plots, histograms, and box plots	5. Use appropriate tools strategically.
Interpreting Categorical and Quantitative Data	**Summarize, represent, and interpret data on a single count or measurement variable.** *In grades 6-8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution or the existence of extreme data points.*	**S.ID.2 – Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.** • Choose the appropriate measure for cent (mean, median) and spread (interquartile range, standard deviation) based on the shape of a data distribution • Use appropriate statistics for center and spread to compare two or more data sets	3. Construct viable arguments and critique the reasoning of others.
Interpreting Categorical and Quantitative Data	**Summarize, represent, and interpret data on a single count or measurement variable.** *In grades 6-8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution or the existence of extreme data points.*	**S.ID.3 – Interpret differences in shape, center and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).** • Define the context of data sets as meaning the specific nature of the attributes under investigation • Interpret differences in shape, center, and spread in the context of the data sets • Describe the possible effects the presence of outliers in a set of data can have on shape, center, and spread in the context of the data sets	2. Reason abstractly and quantitatively. 6. Attend to precision.
Interpreting Categorical and Quantitative Data	**Summarize, represent, and interpret data on two categorical and quantitative variables.** *Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.*	**S.ID.5 – Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.** • Recognize the differences between joint, marginal, and conditional relative frequencies • Summarize categorical data for two categories in two-way frequency tables • Interpret relative frequencies in the context of the data • Recognize possible associations and trends in the data • Calculate relative frequencies including joint, marginal, and conditional	2. Reason abstractly and quantitatively. 4. Model with mathematics. 7. Look for and make use of structure

Algebra 1

Unit 3: Descriptive Statistics

Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Interpreting Categorical and Quantitative Data	**Summarize, represent, and interpret data on two categorical and quantitative variables.** *Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals. S.ID.6b should be focused on linear models, but may be used to preview quadratic functions in Unit 5 of this course.*	**S.ID.6 – Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.** **a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.** **b. Informally assess the fit of a function by plotting and analyzing residuals.** **c. Fit a linear function for a scatter plot that suggests a linear association.** • Represent data on a scatter plot (2 quantitative variables) • Fit a given function class (e.g., linear, exponential) to a data set • Represent the residuals from a function and the data set it models, numerically and graphically • Using given scatter plot data represented on the coordinate plane, informally describe how the two quantitative variables are related • Determine which function best models scatter plot data represented on the coordinate plane, and describe how the two quantitative variables are related • Use functions fitted to data to solve problems in the context of the data	1. Make sense of problems and persevere in solving them. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 7. Look for and make use of structure.
Interpreting Categorical and Quantitative Data	**Interpret linear models.** *Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship. The important distinction between a statistical relationship and a cause-and-effect relationship arise in S.ID.9.*	**S.ID.7 – Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.** • Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data	1. Make sense of problems and persevere in solving them. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 8. Look for and express regularity in repeated reasoning.
Interpreting Categorical and Quantitative Data	**Interpret linear models.** *Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship. The important distinction between a statistical relationship and a cause-and-effect relationship arise in S.ID.9.*	**S.ID.8 – Compute (using technology) and interpret the correlation coefficient of a linear fit.** • Define the correlation coefficient • Interpret the correlation coefficient of a linear fit as a measure of how well the data fit the relationship • Using technology, compute the correlation coefficient of a linear fit	1. Make sense of problems and persevere in solving them. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 7. Look for and make use of structure.

Algebra 1			
Unit 3: Descriptive Statistics			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Interpreting Categorical and Quantitative Data	**Interpret linear models.** *Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship. The important distinction between a statistical relationship and a cause-and-effect relationship arise in S.ID.9.*	**S.ID.9 – Distinguish between correlation and causation.** **• Define positive, negative, and no correlation and explain why correlation does not imply causation** • Define causation • Distinguish between correlation and causation	2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.

Algebra 1			
Unit 4: Expressions and Equations			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Seeing Structure in Expressions	**Interpret the structure of expressions.** *Focus on quadratic and exponential expressions. For A.SSE.1b, exponents are extended from the integer exponents found in Unit 1 to rational exponents focusing on those that represent square or cube roots.*	**A.SSE.1 – Interpret expressions that represent a quantity in terms of its context (Modeling standard).** **a. Interpret parts of an expression, such as terms, factors, and coefficients.** **b. Interpret complicated expressions by viewing one or more of their parts as a single entity, For example, interpret P(1+r)n as the product of P and a factor not depending on P.** • Define and recognize parts of an expression, such as terms, factors, and coefficients • Interpret parts of an expression, such as terms, factors, and coefficients in terms of the context • Interpret complicated expressions, in terms of the context, by viewing one or more of their parts as a single entity	2. Reason abstractly and quantitatively. 4. Model with mathematics. 7. Look for and make use of structure.
Seeing Structure in Expressions	**Interpret the structure of expressions.** *Focus on quadratic and exponential expressions*	**A.SSE.2 – Use the structure of an expression to identify ways to rewrite it. For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2).** • Identify ways to rewrite expressions, such as difference of squares, factoring out a common monomial, and regrouping • Identify various structures of expressions • Use the structure of an expression to identify ways to rewrite it • Classify expressions by structure and develop strategies to assist in classification	1. Make sense of problems and persevere in solving them. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Seeing Structure in Expressions	**Write expressions in equivalent forms to solve problems.** *It is important to balance conceptual understanding and procedural fluency in work with equivalent of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.*	**A.SSE.3 – Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.** **a. Factor a quadratic expression to reveal the zeros of the function it defines.** **b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.** **c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as (1.151/12) 12t j 1.01212t to reveal the approximate equivalent monthly interest rate of the annual rate is 15%.** • Explain the connection between the factored form of a quadratic expression and the zeros of the function it defines • Explain the connection between the completed square form of a quadratic expression and the maximum or minimum value of the function it defines • Explain the properties of the quantity represented by the quadratic expression • Explain the properties of the quantity or quantities represented by the transformed exponential expression • Complete the square on a quadratic expression to produce an equivalent form of an expression • Choose and produce an equivalent form of a quadratic expression to reveal and explain properties of the quantity represented by the original expression • Use the properties of exponents to transform simple expressions for exponential functions • Choose and produce an equivalent form of an exponential expression to reveal and explain properties of the quantity represented by the original expression • Choose and produce an equivalent form of a quadratic expression to reveal and explain properties of the quantity represented by the original expression • Factor a quadratic expression to produce an equivalent form of the original expression	2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics 6. Attend to precision 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Algebra 1			
Unit 4: Expressions and Equations			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Arithmetic with Polynomials and Rational Expressions	**Perform arithmetic operations on polynomials.** *Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of x.*	**A.APR.1 – Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.** • Define closure • Identify that the sum, difference, or product of two polynomials will always be a polynomial, which means that polynomials are closed under the operations of addition, subtraction, and multiplication • Apply arithmetic operations of addition, subtraction, and multiplication to polynomials	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Creating Equations	**Create equations that describe numbers or relationships.** *Extend work on linear and exponential equations in Unit 1 to quadratic equations.*	**A.CED.1 – Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.** **• Describe the relationships between the quantities in the problem (for example, how the quantities are changing or growing with respect to each other); express these relationships using mathematical operations to create an appropriate equation or inequality to solve** • Use all available types of functions to create such equations, including root functions, but constrain to simple cases • Compare and contrast problems that can be solved by different types of equations • Compare and contrast problems that can be solved by different types of equations (linear, exponential) • Solve linear and exponential equations in one variable • Solve inequalities in one variable • Solve all available types of equations and inequalities including root equations and inequalities, in one variable • Create equations and inequalities in one variable and use them to solve problems • Create equations and inequalities in one variable to model real-world situations • Create equations (linear, exponential) and inequalities in one variable and use them to solve problems	1. Make sense of problems and persevere in solving them. 4. Model with mathematics. 8. Look for and express regularity in repeated reasoning.
Creating Equations	**Create equations that describe numbers or relationships.** *Extend work on linear and exponential equations in Unit 1 to quadratic equations.*	**A.CED.2 – Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.** • Identify the quantities in a mathematical problem or real-world situation that should be represented by distinct variables and describe what quantities the variable represent • Graph one or more created equation on coordinate axes with appropriate labels and scales • Justify which quantities in a mathematical problem or real-world situation are dependent and independent of one another and which operations represent those relationships • Determine appropriate units for the labels and scale of graph depicting the relationship between equations created in two or more variables • Create at least two equations in two or more variables to represent relationships between quantities	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision

Algebra 1			
Unit 4: Expressions and Equations			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Creating Equations	**Create equations that describe numbers or relationships.** *Extend work on linear and exponential equations in Unit 1 to quadratic equations. Extend A.CED.4 to formulas involving squared variables.*	**A.CED.4 – Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.** • Define a quantity of interest to mean any numerical or algebraic quantity (e.g., 2(a/b)=d in which 2 is the quantity of interest showing that d must be even; (ϖr2h/3)=Vcone and ϖr2h=Vcylinder showing that Vcylinder=3*Vcone) • Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. (e.g., ϖr2 can be re-written as (ϖr)r which makes the form of this expression resemble Bh. The quantity of interest could also be (a + b)n = an b0 + a(n-1)b1 +...+ a0b n)	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 4. Model with mathematics. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Reasoning with Equations and Inequalities	**Solve equations and inequalities in one variable.** *Students should learn of the existence of the complex nunber system, but will not solve quadratics with complex solutions until Algebra II.*	**A.REI.4 – Solve quadratic equations in one variable.** **a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p)2 = q that has the same solutions. Derive the quadratic formula from this form.** **b. Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.** • Recognize when the quadratic formula gives complex solutions • Derive the quadratic formula by completing the square on a quadratic equation in x • Express complex solutions as a ± bi for real number solutions as a and b • Determine appropriate strategies to solve problems involving quadratic equations, as appropriate to the initial form of the equation • Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p)2 = q that has the same solutions • Solve quadratic equations in one variable	1. Make sense of problems and persevere in solving them. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Reasoning with Equations and Inequalities	**Solve systems of equations.** *Include systems consisting of one linear and one quadratic equation. Include systems that lead to work with fractions. For example, finding the intersections between* $x^2+y^2=1(x=1)/2$ *leads to the point (3/5, 4/5) on the unit circle, corresponding to the Pythagorean triple 3 squared +4 squared=5 squared.*	**A.REI.7 – Solve a simple system consisting of a linear equation and a quadratic equation in two variable algebraically and graphically . For example, find the points of intersection between the line y = -3x and the circle x2 + y2 = 3.** • Transform a simple system consisting of a linear equation and a quadratic equation in two variables so that a solution can be found algebraically and graphically • Explain the correspondence between the algebraic and graphical solutions to a simple system consisting of a linear equation and a quadratic equation in two variables	1. Make sense of problems and persevere in solving them. 8. Look for and express regularity in repeated reasoning.

Algebra 1			
Unit 5: Quadratic Functions and Modeling			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
The Real Number System	**Extend the properties of exponents to rational exponents.** *Connect N.RN.3 to physical situations, e.g., finding the perimeter of a square of area 2.*	**N.RN.3 – Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.** • Find the sums and products of rational and irrational numbers • Recognize that the sum of a rational number and an irrational number is irrational • Recognize that the product of a nonzero rational number and an irrational number is irrational • Explain why rational numbers are closed under addition or multiplication	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 6. Attend to precision. 7. Look for and make use of structure.
Interpreting Functions	**Interpret functions that arise in applications in terms of context.** *Focus on quadratic functions; compare with linear and exponential functions studied in Unit 2.*	**F.IF.4 – For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.** • Define and recognize key features in tables and graphs of linear and exponential functions; intercepts; intervals where the function is increasing, decreasing, positive, or negative, and end behavior • Define and recognize key features in tables and graphs of linear, exponential, and quadratic functions: intercepts; intervals where the function is increasing, decreasing, positive, or negative, relative maximums, symmetries, end behavior and periodicity • Identify the type of function, given a table or graph • Identify whether a function is linear or exponential, given its table or graph • Interpret key features of graphs and tables of functions in terms of the contextual quantities each function represents • Sketch graphs showing the key features of a function, modeling a relationship between two quantities, given a verbal description of the relationship	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 4. Model with mathematics.
Interpreting Functions	**Interpret functions that arise in applications in terms of context.** *Focus on quadratic functions; compare with linear and exponential functions studied in Unit 2.*	**F.IF.5 – Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.** • Identify and describe the domain of a function, given the graph or a verbal/written description of a function • Identify an appropriate domain based on the unit, quantity, and type of function it describes • Relate the domain of a function to its graph and to the quantitative relationship it describes, where applicable • Explain why a domain is appropriate for a given situation	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision.

Algebra 1			
Unit 5: Quadratic Functions and Modeling			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Interpreting Functions	**Interpret functions that arise in applications in terms of context.** *Focus on quadratic functions; compare with linear and exponential functions studied in Unit 2.*	**F.IF.6 – Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.** • Recognize slope as an average rate of change • Estimate the rate of change from a linear or exponential graph • Interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval • Calculate the average rate of change of a function (presented symbolically or as a table) over a specified interval	2. Reason abstractly and quantitatively. 4. Model with mathematics. 5. Use appropriate tools strategically.
Interpreting Functions	**Analyze functions using different representations.** *For F.IF.7b, compare and contrast absolute value, step and piecewise-defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range, and usefulness when examining piecewise-defined functions. Note that this unit, and in particular in F.IF.8b, extends the work begun in Unit 2 on exponential functions with integer exponents. For F.IF.9, focus on expanding the types of functions considered to include linear, exponential, and quadratic. Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored.*	**F.IF.7 – Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.** **a. Graph linear and quadratic functions and show intercepts, maxima, and minima** **b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.** **c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.** **d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.** **e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.** • Determine the difference between simple and complicated polynomial functions • Determine the difference between simple and complicated linear, quadratic, square root, cube root, and piecewise-defined functions • Determine the differences between simple and complicated linear and exponential functions and know when the use of technology is appropriate • Compare and contrast absolute value, step-and piecewise-defined functions with linear, quadratic, and exponential functions • Compare and contrast the domain and range of absolute value, step-and piecewise-defined functions with linear, quadratic, and exponential functions • Compare and contrast the domain and range of exponential, logarithmic, and trigonometric functions with linear, quadratic, absolute value, step-and piecewise-defined functions • Analyze the difference between simple and complicated linear, quadratic, square root, cube root, piecewise-defined, exponential, logarithmic, and trigonometric functions, including step and absolute value functions • Select the appropriate type of function, taking into consideration the key features, domain, and range, to model a real-world situation • Relate the relationship between zeros of quadratic functions and their factored forms to the relationship between polynomial functions of degrees greater than two • Graph exponential functions, by hand in simple cases or using technology for more complicated cases, and show intercepts and end behavior • Graph polynomial functions, by hand in simple cases or using technology for more complicated cases, and show/label maxima and minima of the graph, identify zeros when suitable factorizations are available, and show end behavior • Graph exponential, logarithmic, and trigonometric functions, by hand in simple cases or using technology for more complicated cases. For exponential and logarithmic functions, show: period, midline, and amplitude • Graph linear functions by hand in simple cases or using technology for more complicated cases and show/label intercepts of the graph • Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions, by hand in simple case or using technology for more complicated cases, and show/label key features of the graph	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision.

Algebra 1			
Unit 5: Quadratic Functions and Modeling			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Interpreting Functions	**Analyze functions using different representations**. *For F.IF.7b, compare and contrast absolute value, step and piecewise-defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range, and usefulness when examining piecewise-defined functions. Note that this unit, and in particular in F.IF.8b, extends the work begun in Unit 2 on exponential functions with integer exponents. For F.IF.9, focus on expanding the types of functions considered to include linear, exponential, and quadratic. Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored.*	**F.IF.8 – Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.** **a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.** **b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t y = (1.01)12t, y = (1.2)/10, and classify them as representing exponential growth or decay.** • Identify different forms of a quadratic expression • Identify zeros, extreme values, and symmetry of the graph of a quadratic function • Identify how key features of a quadratic function relate to characteristics of a real-world context • Classify the exponential function as exponential growth or decay by examining the base • Identify how key features of an exponential function relate to tis characteristics in a real-world context • Interpret different yet equivalent forms of a function, as defined by an expression in terms of • Use the properties of exponents to interpret expressions for exponential functions in a real-world context • Given the expression of an exponential function, interpret the expression in terms of a real-world context, using the properties of exponents • Given the expression of a quadratic function, interpret zeros, extreme values, and symmetry of the graph in terms of a real-world context • Write a quadratic function defined by an expression in different but equivalent forms to reveal and explain various properties of the function and determine which form of the quadratic is the most appropriate for showing zeros and symmetry of a graph in terms of a real-world context • Write an exponential function defined by an expression in different but equivalent forms to reveal and explain different properties of the function, and determine which form of the function is the most appropriate for interpretation in a real-world context • Write functions in equivalent forms using the process of factoring • Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure.

Algebra 1			
Unit 5: Quadratic Functions and Modeling			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Interpreting Functions	**Analyze functions using different representations.** *For F.IF.7b, compare and contrast absolute value, step and piecewise-defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range, and usefulness when examining piecewise-defined functions. Note that this unit, and in particular in F.IF.8b, extends the work begun in Unit 2 on exponential functions with integer exponents. For F.IF.9, focus on expanding the types of functions considered to include linear, exponential, and quadratic. Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored.*	**F.IF.9 – Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.** • Identify types of functions based on verbal, numerical, algebraic, and graphical descriptions and state key properties • Differentiate between exponential and linear functions using a variety of descriptors (graphical, verbal, numerical, algebraic) • Differentiate between different types of functions using a variety of descriptors (graphical, verbal, numerical, algebraic) • Use a variety of function representations (algebraic, graphical, numerical in tables, or by verbal descriptions) to compare and contrast properties of two functions	1. Make sense of problems and persevere in solving them. 5. Use appropriate tools strategically. 7. Look for and make use of structure.
Building Functions	**Build a function that models a relationship between two quantities.** *Focus on situations that exhibit a quadratic relationship.*	**F.BF.1 – Write a function that describes a relationship between two quantities.** **a. Determine an explicit expression, a recursive process, or steps for calculation from a context.** **b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.** **c. (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.** • Define explicit function and recursive process • Combine two functions using the operations of addition, subtraction, multiplication, and division • Evaluate the domain of the combined function • Given a real-world situation or mathematical problem, build standard functions to represent relevant relationships/quantities • Given a real-world situation or mathematical problem, determine which arithmetic operation should be performed to build the appropriate combined function • Given a real-world situation or mathematical problem, relate the combined function to the context of the problem • Write a function that describes a relationship between two quantities by determining an explicit expression, a recursive process, or steps for calculation from a context	2. Reason abstractly and quantitatively. 4. Model with mathematics. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Algebra 1			
Unit 5: Quadratic Functions and Modeling			
Domains	Clusters with Instructional Notes	Montana Common Core Standards with Billings Public Schools Deconstructed Learning Objectives	Mathematical Practices
Building Functions	**Build new functions from existing functions.** *For F.BF.3, focus on quadratic functions, and consider including absolute value functions.*	**F.BF.3– Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.** • Given a single transformation on a function symbolic or graphic identify the effect on the graph • Using technology, identify effects of single transformations on graphs of functions • Recognize even and odd functions from their graphs and equations • Describe the differences and similarities between a parent function and the transformed function • Find the value of k, given the graphs of a parent function, f(x), and the transformed function; f(x) + k, k f(x), f(kx), or f(x + k) • Experiment with cases and illustrate an explanation of the effects on a graph, using technology • Graph a given function by replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative)	4. Model with mathematics. 5. Use appropriate tools strategically. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Building Functions	**Build new functions from existing functions.** *For B.BF.4a, focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as $f(x)=x^2$, x>0.*	**F.BF.4– Find the inverse functions.** **a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example: f(x) = 2x3 or fx = (x+1)/(x-1) for x ≠ 1.** **b. (+) Verify by composition that one function is the inverse of another.** **c. (+) Read values of an inverse function from a graph or a table, given the function has and inverse.** **d. (+) Produce an invertible function from a non-invertible function by restricting the domain.** • Define inverse function • Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse	4. Model with mathematics. 5. Use appropriate tools strategically. 7. Look for and make use of structure.
Linear, Quadratic, and Exponential Models	**Construct and compare linear, quadratic, and exponential models and solve problems.** *Compare linear and exponential growth to quadratic growth.*	**F.LE.3 – Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.** • Informally define the concept of end behavior • Fluently compute growth rates for linear, exponential, and quadratic functions • Compare tables and graphs of linear and exponential functions to observe that a quantity, increasing exponentially, exceeds all others to solve mathematical and real-world problems • Compare tables and graphs of exponential and other polynomial functions to observe that a quantity, increasing exponentially, exceeds all others to solve mathematical and real-world problems	2. Reason abstractly and quantitatively. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 8. Look for and express regularity in repeated reasoning.

GLOSSARY

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (– 3/4) = (– 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction *A*/*B* where *A* and/or *B* are fractions (*B* nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by *counting on*—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. *See:* line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

First quartile. For a data set with median *M*, the first quartile is the median of the data values less than *M*. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[2] *See also:* median, third quartile, interquartile range.

[1] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.
[2] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," *Journal of Statistics Education* Volume 14, Number 3 (2006).

Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.) *See also:* rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Integer. A number expressible in the form a or $-a$ for some whole number a.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also:* first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.4 Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 × 4/3 = 4/3 × 3/4 = 1.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

[3]Adapted from Wisconsin Department of Public Instruction, *op. cit.*
[4]To be more precise, this defines the *arithmetic mean.* one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space.

Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form a/b or $-a/b$ for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of

Repeating decimal. The decimal form of a rational number. *See also:* terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of Bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[5]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3, . . .

[5]Adapted from Wisconsin Department of Public Instruction, *op. cit.*

Tables

Table 1. Common addition and subtraction situations.[1]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[2]**
Put Together/ Take Apart[1]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, 5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[2]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5, 5 - 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?, 3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?, ? + 3 = 5$

[1]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

[1]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

Table 2. Common multiplication and division situations.[1]

	Unknown Product $3 \times 6 = ?$	Group Size Unknown ("How many in each group?" Division) $3 \times ? = 18$, and $18 \div 3 = ?$	Number of Groups Unknown ("How many groups?" Division) $? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[1]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3. The properties of operations. **Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.**

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4. The properties of equality. **Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.**

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$
Addition property of equality	If $a = b$, then $a + c = b + c$
Subtraction property of equality	If $a = b$, then $a - c = b - c$
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality. **Here a, b and c stand for arbitrary numbers in the rational or real number systems.**

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.

If $a > b$ and $b > c$ then $a > c$.

If $a > b$, then $b < a$.

If $a > b$, then $-a < -b$.

If $a > b$, then $a \pm c > b \pm c$.

If $a > b$ and $c > 0$, then $a \times c > b \times c$.

If $a > b$ and $c < 0$, then $a \times c < b \times c$.

If $a > b$ and $c > 0$, then $a \div c > b \div c$.

If $a > b$ and $c < 0$, then $a \div c < b \div c$.

LEARNING PROGRESSIONS BY DOMAIN

<table>
<tr><th colspan="10">Mathematics Learning Progressions by Domain</th></tr>
<tr><th>K</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>HS</th></tr>
<tr><td>Counting and Cardinality</td><td colspan="8"></td><td rowspan="3">Number and Quantity</td></tr>
<tr><td colspan="6">Number and Operations in Base Ten</td><td colspan="3">Ratios and Proportional Relationship</td></tr>
<tr><td colspan="3"></td><td colspan="3">Number and Operations – Fractions</td><td colspan="3">The Number System</td></tr>
<tr><td colspan="6" rowspan="2">Operations and Algebraic Thinking</td><td colspan="3">Expressions and Equations</td><td>Algebra</td></tr>
<tr><td colspan="2"></td><td colspan="2">Functions</td></tr>
<tr><td colspan="10">Geometry</td></tr>
<tr><td colspan="6">Measurement and Data</td><td colspan="4">Statistics and Probability</td></tr>
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